

Assignment 3 (Due: Dec. 31, 2024)

1. **(Math)** Nonlinear least-squares. Suppose that $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_m(\mathbf{x})) : \mathbb{R}^n \rightarrow \mathbb{R}^m$, $\mathbf{x} \in \mathbb{R}^n, \mathbf{f} \in \mathbb{R}^m$ and some $f_i(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}$ is a (are) non-linear function(s). Then, the problem,

$$\mathbf{x}^* = \arg \min_{\mathbf{x}} \frac{1}{2} \|\mathbf{f}(\mathbf{x})\|_2^2 = \arg \min_{\mathbf{x}} \frac{1}{2} (\mathbf{f}(\mathbf{x}))^T \mathbf{f}(\mathbf{x})$$

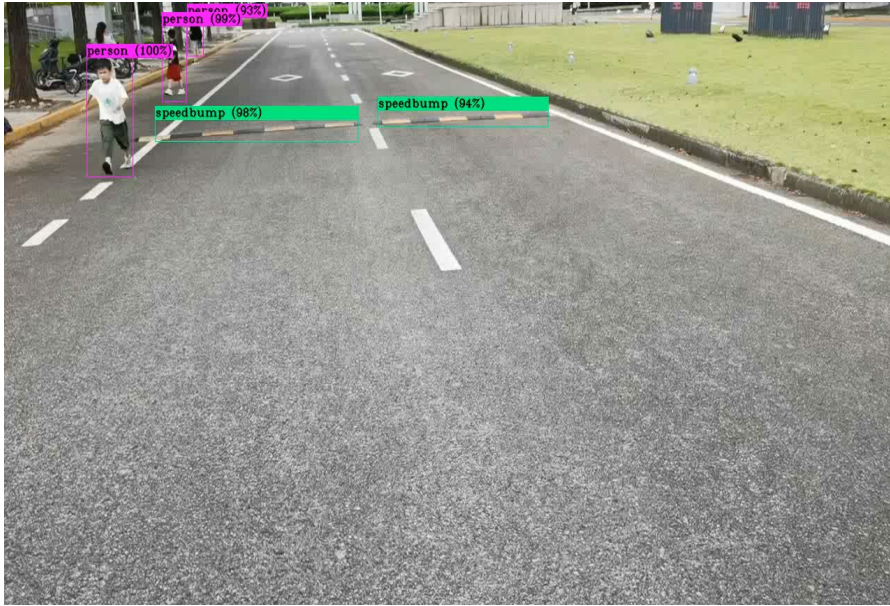
is a nonlinear least-squares problem. In our lecture, we mentioned that Levenberg-Marquardt algorithm is a typical method to solve this problem. In L-M algorithm, for each updating step, at the current \mathbf{x} , a local approximation model is constructed as,

$$\begin{aligned} L(\mathbf{h}) &= \frac{1}{2} (\mathbf{f}(\mathbf{x} + \mathbf{h}))^T \mathbf{f}(\mathbf{x} + \mathbf{h}) + \frac{1}{2} \mu \mathbf{h}^T \mathbf{h} \\ &= \frac{1}{2} (\mathbf{f}(\mathbf{x}))^T \mathbf{f}(\mathbf{x}) + \mathbf{h}^T (\mathbf{J}(\mathbf{x}))^T \mathbf{f}(\mathbf{x}) + \frac{1}{2} \mathbf{h}^T (\mathbf{J}(\mathbf{x}))^T \mathbf{J}(\mathbf{x}) \mathbf{h} + \frac{1}{2} \mu \mathbf{h}^T \mathbf{h} \end{aligned}$$

where $\mathbf{J}(\mathbf{x})$ is $\mathbf{f}(\mathbf{x})$'s Jacobian matrix, and $\mu > 0$ is the damped coefficient. Please prove that $L(\mathbf{h})$ is a strictly convex function. (Hint: If a function $L(\mathbf{h})$ is differentiable up to at least second order, L is strictly convex if its Hessian matrix is positive definite.)

2. **(Programming)** I have established a dataset for training models for detecting speed-bumps and persons. This dataset can be downloaded from <https://github.com/csLinZhang/CVBook/tree/main/chapter-15-YOLO/For-yolov4>. Using this dataset, please train a speed-bump detection model and test your model on the provided test video (on the course website). For this question, you only need to hand in your video with detected bounding-boxes to the TA. A sample

frame of our result video may like the following image.



3. **(Experiment)** 3D face scan and editing. Please refer to the files on the course website.