SR-LLA: A NOVEL SPECTRAL RECONSTRUCTION METHOD BASED ON LOCALLY LINEAR APPROXIMATION

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ABSTRACT

Compared with tristimulus, spectrum contains much more information of a color, which can be used in many fields, such as disease diagnosis and material recognition. In order to get an accurate and stable reconstruction of spectral data from a tristimulus input, a method based on locally linear approximation is proposed in this paper, namely SR-LLA. To test the performance of SR-LLA, we conduct experiments on three Munsell databases and present a comprehensive analysis of its accuracy and stability. We also compare the performance of SR-LLA with the other two spectral reconstruction methods based on BP neural network and PCA, respectively. Experimental results indicate that SR-LLA could outperform other competitors in terms of both accuracy and stability for spectral reconstruction.

Index Terms—Spectral reconstruction, Munsell dataset, locally linear approximation

1. INTRODUCTION

Traditionally, we use tristimulus values to define or express a color, such as CIE-XYZ, due to the limitation of display devices. However, the tristimulus system of color loses great amount of color information. In contrast, the spectral data can store more useful information for color reproduction as well as for color specification under different viewing conditions, which is widely used in many fields [1-4].

So far, there have been quite a few algorithms proposed for spectral reconstruction. Usui *et al*. constructed a five-layer BP neural network and generated an identity mapping of the surface spectral-reflectance data of 1280 Munsell color chips, using a back-propagation learning algorithm [5]. Fairman *et al.* applied PCA to several databases [6-8]. In 2006, according to the ten hues fitted to Munsell database by Cohen [9], Ayala *et al.* divided the color space into ten zones [10]. With three eigenvectors, the principal components of the color can be obtained, which can be used to produce spectral data. Abed *et al*. proposed a linear interpolation method to reconstruct color information from the corresponding colorimetric values under a given set of viewing conditions [11]. To improve the color reproduction accuracy of spectral images, Zhang *et al*. raised two novel interim connection spaces [12]. According to the spectral and colorimetric representing accuracy of Munsell and Natural Systems chips, these two connection spaces were believed to be better than LabPQR and the ICS with two sets of tristimulus under two real light sources.

Actually, the tristimulus color space can be viewed as a low dimensional manifold while its corresponding spectral data can be regarded as a high dimensional manifold. In the literature of manifold analysis, there are already some methods proposed to establish a mapping between the low and the corresponding high dimensional manifolds [14, 15]. Inspired by these ideas, in this paper, we propose a novel method for Spectral Reconstruction based on Locally Linear Approximation, namely SR-LLA. Accuracy and stability of the proposed SR-LLA are corroborated by the experiments conducted on benchmark datasets.

The remainder of this paper is organized as follows. Section 2 presents our proposed SR-LLA spectral reconstruction algorithm. Section 3 reports the experimental results while section 4 concludes the paper.

2. SPECTRAL RECONSTRUCTION BASED ON LOCALLY LINEAR APPROXIMATION

2.1. Relationship between color spectra and tristimulus data

The color matching functions are the numerical description of the chromatic response, which converts the spectral color into CIE-XYZ data. These functions are defined as follows,

$$\begin{aligned} X &= t \int P(\lambda)\bar{x} d_\lambda \\ Y &= t \int P(\lambda)\bar{y} d_\lambda \\ Z &= t \int P(\lambda)\bar{z} d_\lambda , \end{aligned} \tag{1}$$

where $\bar{x}$, $\bar{y}$, and $\bar{z}$ represent the eye's sensitivity to brightness and $P(\lambda)$ is the spectral data, t is a self-luminous body, such as CRT, which is equal to 680 lumens per watt in our experiments.

For two tristimulus vectors $[X_1 \quad Y_1 \quad Z_1]^T$ and $[X_2 \quad Y_2 \quad Z_2]^T$, since $\bar{x}$, $\bar{y}$, $\bar{z}$ and t are nonnegative constants and maintain the same under the determined illuminant and observer, when the gap between them tends to zero, $P(\lambda)_1 - P(\lambda)_2$ also tends to zero. In other words, when two tristimulus vectors in the three dimensional space are close

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enough, the corresponding spectral vectors in high dimensional space are also close.

For most applications, values are calculated at 10nm intervals instead of continuous functions. So Eq. (1) can be rewritten as

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = MP(\lambda), \tag{2}$$

where M is a combination of $t\bar{x}$, $t\bar{y}$, and $t\bar{z}$ on each wavelength, and $P(\lambda)$ is the spectral vector with 31 elements. Eq. (2) will be used for calculation in this paper.

Reconstructing a high dimensional spectral point from a tristimulus input is actually an inverse mapping of the color matching functions. However, as the color matching function is nonlinear, it is nearly impossible to find a direct solution of spectral reconstruction. To this end, we attempt to design an efficient and accurate approach, named *Spectral Reconstruction based on Locally Linear Approximation*, SR-LLA for short.

2.2. SR-LLA algorithm

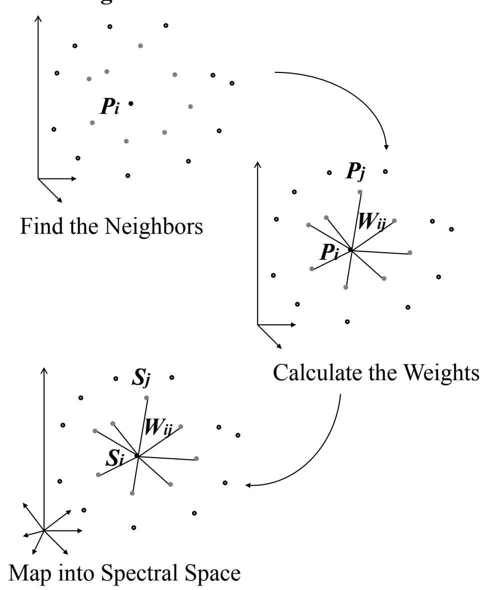

Fig. 1. Pipeline of the proposed spectral reconstruction algorithm SR-LLA.

Knowing that the local linearity in the tristimulus space is kept in the spectral space, any spectral color can be approximated through weighted linear combination of similar colors. The local weights in the spectral space are the same as the ones in the tristimulus space. Based on this idea, we propose a novel method for spectral reconstruction based on locally linear approximation, namely SR-LLA. Such a method is essentially an unsupervised learning method, which means that it does not need the training procedure before reconstruction. As it is well known that Munsell dataset contains more than one thousand colors with both spectral and tristimulus data [16], the relationship of nonlinear mapping from tristimulus to the spectral space has been embodied in Munsell dataset. In our method, Munsell dataset with both spectral and tristimulus data will be used in SR-LLA to learn the mapping function.

The proposed method SR-LLA borrows the basic idea from manifold learning methods [14] and is mainly composed of three steps, as illustrated in Fig. 1.

In the first step, each pixel in the color image with tristimulus values P_i is considered as a point in the three dimensional space, and its k nearest neighbors are picked from the Munsell colors in the same space. The number k of the neighbors is a free parameter to preset. Usually, more neighbors can improve the quality of the reconstruction, while adding the computational complexity of the algorithm.

The second step will find the optimal weights W_{ij} of k neighbors for best approximating P_i. To gain the best combination of weights, we define a cost function $C(W)$ as the sum of the squared difference between a pixel and its approximation $\sum_j W_{ij} P_j$:

$$C(W) = \sum_i \left\| P_i - \sum_j W_{ij} P_j \right\|^2, \tag{3}$$

where the sum of weights should always be equal to 1. Note that there could be negative numbers in the group of W_{ij}. By minimizing the cost function $C(W)$ in Eq. (3), the approximation error can reach the least. This minimization problem can be solved with the least-squares method.

Given the spectral values of the Munsell colors, the corresponding tristimulus is easily computed as discussed in Section 2.1. Assuming that the local linearity of colors is maintained in different color spaces, the weights W_{ij} computed with Eq. (3) in the tristimulus Munsell color space can be directly applied to the spectral Munsell color space for reproducing the color spectra. Therefore, in the third step of the proposed algorithm, color spectra S_i can be approximated as

$$S_i = \sum_1^k W_{ij} S_{ij}, \tag{4}$$

where S_{ij} is one of the nearest neighbors of S_i in the spectral Munsell color space. The neighborhood relationship and the weights W_{ij} are respectively inherited from the first and second steps. The proposed SR-LLA algorithm is briefly summarized in Table 1.

Table 1. The spectral reconstruction algorithm SR-LLA

Input:	a color image with tristimulus values P_i, the number k of nearest neighbors and the Munsell dataset
1.	select k nearest neighbors from the Munsell colors for each pixel in the input space;
2.	calculate the weights W_{ij} through minimizing the cost function (3);
3.	apply the weights W_{ij} to the corresponding spectral Munsell colors and compute the color spectra for all pixels;
Output:	a spectral color image with color spectra S_i

3. EXPERIMENTAL RESULTS

3.1. Experiment data

Five images are tested in our experiments: Woman, Girl, Leaves, Grassland and Tree Bark, as shown in Fig. 2. These images and the associated spectral data can be downloaded from [17]. Three Munsell database, MATT (spectrofotometer measured), MATT (AOTF measured), Glossy (spectrofotometer measured) [9], are used as a reference set to estimate the color spectra of the tested images. CIE 1964 supplementary standard colorimetric observer was employed.

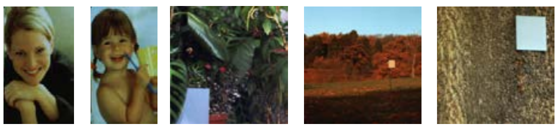

Fig. 2. Five input images.

Tristimulus values in CIE-XYZ space are respectively employed for spectral reconstruction. For Munsell dataset, the spectral and tristimulus values are known and can be downloaded from the website [18]. We used the 31 dimension spectral data which ranged from 400nm to 700nm and had an interval of 10nm. The three dimension values, both XYZ and Lab, are calculated with different illuminant.

The SR-LLA algorithm and other two algorithms for comparison were employed for reconstructing the spectral data of five tested images.

The reconstruction error can be defined as the average of the absolute errors between all real and reconstructed color spectra on each pixel,

$$E_{ij} = \frac{|O_{ij} - R_{ij}|}{D}, \quad (5)$$

where E_{ij} stands for the error on the pixel of row i and column j, and O and R means the original spectra and reconstructed spectra. D denotes the dimension of the spectra. A smaller error stands for a better reconstruction.

3.2. Database comparison

Table 2 shows the statistical results of the spectral reconstruction in three Munsell datasets. The color coordinates were calculated with CIE64 Observer and CIE Illuminant D65. The number of neighbors k was set as 200.

From Table 2 we can find that the reconstruction with MATT (spectrofotometer measured), which has 1269 specimens, was almost the same with the one with MATT (AOTF measured), which has 1250 specimens, in both spectral space and CIE-Lab space. But with 1600 specimens of Glossy (spectrofotometer measured), the reconstruction was not as good as the former two.

Looking for the reason why a large database causes a poor result, we checked the distribution of the specimens in different databases. Fig. 3 shows 100 randomly selected specimens from MATT (spectrofotometer measured) and Glossy databases. It's not hard to find that the color distribution was not even in Glossy database. So in some hues, the color distribution is dense, while in others, it is sparse. When the vectors around the reconstructed one are too sparse, the result will be unsatisfactory.

Table 2. Experimental results in different Munsell databases

Munsell MATT (3pectrophotometer measured) 1269					
	Girl	Grassland	Leaves	TreeBart	Woman
mean	0.0210	0.0096	0.0064	0.0100	0.0181
max	0.4724	0.0702	0.1364	0.0640	0.0409
var.	0.0001	0.0001	0.0001	0.0001	0.0001
>%3	0.0002	0.0000	0.0000	0.0000	0.0002
Lab mean	0.0333	0.0250	0.0145	0.0120	0.0226
Lab var.	0.0010	0.0032	0.0023	0.0025	0.0004
Munsell MATT (AOTF measured) 1250					
	Girl	Grassland	Leaves	TreeBart	Woman
mean	0.0191	0.0115	0.0071	0.0087	0.0173
max	0.4737	0.0655	0.1355	0.0617	0.0389
var.	0.0001	0.0001	0.0001	0.0000	0.0001
>%3	0.0002	0.0000	0.0000	0.0000	0.0002
Lab mean	0.0319	0.0236	0.0185	0.0078	0.0316
Lab var.	0.0014	0.0038	0.0031	0.0008	0.0009
Munsell Glossy (spectrofotometer measured) 1600					
	Girl	Grassland	Leaves	TreeBart	Woman
mean	0.0467	0.0167	0.0179	0.0338	0.0299
max	0.5257	0.0675	0.1119	0.1167	0.0700
var.	0.0007	0.0002	0.0004	0.0002	0.0003
>%3	0.0002	0.0000	0.0000	0.0000	0.0002
Lab mean	0.2398	0.0936	0.1792	0.0605	0.2361
Lab var.	0.0577	0.0567	0.1916	0.0213	0.0093

[a](>%3 is the percentage of specimens with color differences greater than 3 units)

[b](Lab mean and Lab var. stand for mean value and variance calculated in CIE-Lab space)

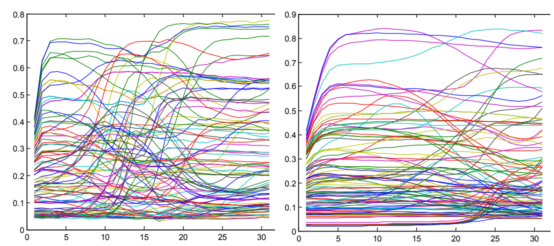

Fig. 3. 100 random specimens in MATT 1269 database (left) and Glossy 1600 database (right).

3.3. Illuminants comparison

Table 3 presents the statistics of spectral reconstruction errors under different CIE Illuminants D65, D75, D50, A and F11. For reconstruction, the tristimulus colors of Munsell were also calculated in different illuminants respectively. The number of neighbors k was set as 200, and the database was Munsell MATT 1269.

Results from D65, D75 and D50 were similar. But the ones from Illuminant A have a larger average error and variance, while the maximum error was not always the largest. This shows that the degree of dispersion is larger than the others. As for Illuminant F11, the average error is not large but the maximum error is usually the largest. However, the variance is not much larger than the results of D65, D75 and D50, which means the result is relatively centralized.

After checking the result carefully, we found that when using Illuminant A, the spectral reconstruction errors for blue and yellow-green were significantly greater than for other colors. However, in the result of D65, the error was rather even. So it was not be brought by the database. The characteristic of Illuminant A was the stress of warm color, so the expression of cold color, like blue, was weakened.

As for the result of Illuminant F11, the larger error was caused by the area of red and yellow-red. This can also be explained with the stress of colder area and the weakness of warmer area of the illuminant.

The extreme large errors (the ones have an error larger than 3% of the original spectra) are the same in the five results of different illuminants.

Table 3. Experiment results under different illuminants

Illuminant D65					
	Girl	Grassland	Leaves	TreeBart	Woman
mean	0.0210	0.0096	0.0064	0.0100	0.0181
max	0.4724	0.0702	0.1364	0.0640	0.0409
var.	0.0001	0.0001	0.0001	0.0001	0.0001
>3%	0.0002	0.0000	0.0000	0.0000	0.0002
Illuminant D75					
	Girl	Grassland	Leaves	TreeBart	Woman
mean	0.0220	0.0097	0.0064	0.0101	0.0189
max	0.4740	0.0694	0.1371	0.0652	0.0430
var.	0.0001	0.0001	0.0001	0.0001	0.0001
>3%	0.0002	0.0000	0.0000	0.0000	0.0002
Illuminant D50					
	Girl	Grassland	Leaves	TreeBart	Woman
mean	0.0183	0.0092	0.0066	0.0097	0.0162
max	0.4686	0.0668	0.1304	0.0665	0.0362
var.	0.0001	0.0001	0.0001	0.0001	0.0001
>3%	0.0002	0.0000	0.0000	0.0000	0.0002
Illuminant A					
	Girl	Grassland	Leaves	TreeBart	Woman
mean	0.0285	0.0341	0.0216	0.0387	0.0244
max	0.1384	0.0766	0.1004	0.1137	0.0548
var.	0.0003	0.0003	0.0002	0.0002	0.0002
>3%	0.0002	0.0000	0.0000	0.0000	0.0002
Illuminant F11					
	Girl	Grassland	Leaves	TreeBart	Woman
mean	0.0138	0.0118	0.0086	0.0153	0.0122
max	0.5655	0.0794	0.1614	0.1281	0.0268
var.	0.0001	0.0001	0.0001	0.0002	0.0001
>3%	0.0002	0.0000	0.0000	0.0000	0.0002

[a](>%3 is the percentage of specimens with color differences greater than 3 units)

3.4. Algorithms comparison

We also compared the performance of the proposed algorithm SR-LLA with the other two spectral reconstruction methods based on BP neural network [5] and PCA trained on Munsell MATT 1269 divided into 10 hues [10], respectively. The results are shown in Table 4.

As a five-layer neural network used for the training starts from and ends with spectral data, the half of the five layers, a three-layer BP neural network, will be studied in this work. The input layer is the tristimulus values of the 1269 Munsell colors and the output layer is the color spectra. The hidden layer is composed of ten nodes. The trained variable of BP neural network was less than 1%.

The dividing of hue was according to Appendix 1 from [10]. The experimental environment remained the same and the results are listed in Table 4.

From Table 4 we can find that the result of SR-LLA was better than those of the other two approaches. In terms of the mean error, the result of SR-LLA was much smaller than the ones of the other two, which shows that the proposed method was the most accurate one. Also, the smaller max error and variance of error proved that the data reconstructed by SR-LLA were more stable than those reconstructed by using the other two methods.

Table 4. Experiment results from different approaches

SR-LLA					
	Girl	Grassland	Leaves	TreeBart	Woman
mean	0.0210	0.0096	0.0064	0.0100	0.0181
max	0.4724	0.0702	0.1364	0.0640	0.0409
var.	0.0001	0.0001	0.0001	0.0001	0.0001
>3%	0.0002	0.0000	0.0000	0.0000	0.0002
Three-layer BP Neural Network					
	Girl	Grassland	Leaves	TreeBart	Woman
mean	0.0241	0.0379	0.0222	0.0306	0.0220
max	0.4447	0.1999	0.2027	0.3097	0.0973
var.	0.0003	0.0018	0.0004	0.0034	0.0002
>3%	0.0002	0.0440	0.0929	0.0006	0.0043
PCA on Divided Munsell Database					
	Girl	Grassland	Leaves	TreeBart	Woman
mean	0.0890	0.0402	0.0442	0.0820	0.0806
max	0.6109	0.1489	0.2309	0.2277	0.1716
var.	0.0025	0.0007	0.0010	0.0011	0.0028
>3%	0.0001	0.0000	0.0000	0.0000	0.0000

4. DISCUSSIONS AND CONCLUSIONS

In this paper, we proposed a novel color spectral reconstruction method inspired by the ideas in the manifold learning field and it is named as SR-LLA. SR-LLA learns the mapping function from the tristimulus space to the high dimensional spectral space based on Munsell database. The experiments show that it is accurate as well as stable.

The reconstructed result depends on several factors, database, illuminant and the number of neighbors. When the database is sparse or partially dense, the result will be impacted and has a larger error. But in a dense and uniform dataset, the result will be satisfactory. Illuminant will also affect the reconstruction. The result obtained under natural daylight illuminant like D65 is better than the ones obtained by using illuminants like A and F11, which prejudicially emphasize on cold or warm colors. Accuracy and stability of the proposed SR-LLA algorithm were verified by the experiments conducted on the benchmark datasets.

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